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MONETARY AND FISCAL POLICY

ELECTIVE COURSE

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2023-2024 Edition

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Monetary and fiscal policy

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2024

1 Relationship between interest rate and the price of a bond

$$B_t(1 + i_t) = 100 \tag{1.1}$$

$$B_t = \frac{100}{1 + i_t} \tag{1.2}$$

2 Relationship between short and long term interest rates

$$x(1 + i_{t,2})^2 = x(1 + i_t)(1 + i^e_{t+1/t}) \tag{2.1}$$

$$(1 + i_{t,2})^2 = (1 + i_t)(1 + i^e_{t+1/t}) \tag{2.2}$$

$$1 + 2i_{t,2} + i^2_{t,2} = 1 + i_t + i^e_{t+1/t} + i_t \cdot i^e_{t+1/t} \tag{2.3}$$

$$2i_{t,2} = i_t + i^e_{t+1/t} \tag{2.4}$$

$$i_{t,2} = \frac{i_t + i^e_{t+1/t}}{2} \tag{2.5}$$

3 Nominal and real interest rates

$$C_t P_t = 100 \tag{3.1}$$

$$C_t = \frac{100}{P_t} \tag{3.2}$$

$$E(C_{t+1}P_{t+1}/\Omega_t) = 100(1 + i_t) \tag{3.3}$$

$$E(C_{t+1}P_{t+1}/\Omega_t) = E(C_{t+1}/\Omega_t)E(P_{t+1}/\Omega_t) \tag{3.4}$$

(3.3) becomes

$$\Leftrightarrow C^e_{t+1/t} = \frac{100(1+i_t)}{P^e_{t+1/t}} \tag{3.5}$$

$$\Leftrightarrow \frac{C^e_{t+1/t} - C_t}{C_t} = \frac{\frac{100(1+i_t)}{P^e_{t+1/t}} - \frac{100}{P_t}}{\frac{100}{P_t}} \tag{3.6}$$

The left term is the percentage difference in consumption, we call it the real interest rate, and we denote it by r_t

$$\Leftrightarrow r_t = \frac{\frac{100(1+i_t)}{P^e_{t+1/t}} - \frac{100}{P_t}}{\frac{100}{P_t}} \cdot \frac{P_t}{P_t} \tag{3.7}$$

$$\Leftrightarrow r_t = \frac{\frac{100(1+i_t)P_t}{P^e_{t+1}}}{100} - 100 \tag{3.8}$$

$$\Leftrightarrow r_t = \frac{P_t}{P^e_{t+1}}(1+i_t) - 1 \tag{3.9}$$

$$\Leftrightarrow 1 + r_t = \frac{P_t}{P^e_{t+1}}(1+i_t) \tag{3.10}$$

We define the expected inflation rate between t and $t+1$

$$\pi^e_{t+1/t} = \frac{P^e_{t+1}}{P_t} - 1 \tag{3.11}$$

From (3.10) follows

$$1 + r_t = \frac{1+i_t}{1+\pi^e_{t+1/t}} \tag{3.12}$$

$$\Leftrightarrow 1 + i_t = (1+r_t)(1+\pi^e_{t+1/t}) \tag{3.13}$$

Using the approximation $r_t \pi^e_{t+1/t} \approx 0$

$$1 + i_t = 1 + r_t + \pi^e_{t+1/t} \tag{3.14}$$

Hence

$$r_t = i_t - \pi^e_{t+1/t} \tag{3.15}$$

4 The IS curve

$$Y^d = C + I + G \tag{4.1}$$

$$C = c_0 + c(Y - T) \tag{4.2}$$

$$I = d_0 + d_1 Y - d_2 i \tag{4.3}$$

G is exogenus. From (4.1)

$$Y^d = c_0 + c(Y - T) + d_0 + d_1 Y - d_2 i + G \tag{4.4}$$

The supply of goods is GDP, Y. Hence

$$Y = c_0 + c(Y - T) + d_0 + d_1 Y - d_2 i + G \tag{4.5}$$

$$\Leftrightarrow Y = c_0 + cY - cT + d_0 + d_1 Y - d_2 i + G \tag{4.6}$$

$$\Leftrightarrow Y = Y(c + d_1) + c_0 - cT + d_0 - d_2 i + G \tag{4.7}$$

$$\Leftrightarrow Y(1 - c - d_1) = c_0 - cT + d_0 - d_2 i + G \tag{4.8}$$

$$\Leftrightarrow Y = \frac{c_0 + d_0 - cT - d_2 i + G}{1 - c - d_1} \tag{4.9}$$

Where we have the multiplier

$$m = \frac{1}{1 - c - d_1} > 1 \tag{4.10}$$

Suppose an increase in G, which is exogenous. What is the impact of this change on Y?

$$\Delta Y = \Delta G[1 + (c + d_1) + (c + d_1)^2 + (c + d_1)^3 + ... + (c + d_1)^n] \tag{4.11}$$

Note that this is a geometric progession that converge to $\frac{a}{1+r}$. In our case $a = 1$ and $r = c + d_1$.

$$\Delta Y = \Delta G \frac{1}{1 - c - d_1} \tag{4.12}$$

$$\Delta Y = \frac{\Delta G}{1 - c - d_1} > \Delta G \tag{4.13}$$

5 The money supply process

$$\Delta R = \sum_{i=0}^{\infty}(1-r)^i \tag{5.1}$$

We know that the series $\sum_{i=0}^{\infty} x^i$ converge to $\frac{1}{1-x}$ if $x < 1$. As $1 - r < 1$

$$\Delta R = \frac{1}{1-(1-r)} \tag{5.2}$$

$$\Leftrightarrow \Delta R = \frac{1}{r} \tag{5.3}$$

The same for Deposits

$$\Delta D = \Delta C \sum_{i=0}^{\infty}(1-r)^i \tag{5.4}$$

$$\Delta D = \Delta C \frac{1}{1-(1-r)} \tag{5.5}$$

$$\Leftrightarrow \Delta D = \frac{\Delta C}{r} \tag{5.6}$$

6 The money multiplier

$$m = \frac{M}{MB} \tag{6.1}$$

In this world base money is made only of reserves: $MB = R = rD$ and broad money is made only of deposits: $M = D$. Hence

$$m = \frac{D}{R} \Leftrightarrow m = \frac{D}{rD} \Leftrightarrow m = \frac{1}{r} \tag{6.2}$$

Now assume that the public does want to hold money, hence $c > 0$

$$MB = R + C \Leftrightarrow MB = rD + cD \Leftrightarrow MB = D(r+c) \Leftrightarrow D = \frac{MB}{r+c} \tag{6.3}$$

$$M = C + D \Leftrightarrow M = cD + D \Leftrightarrow M = D(1+c) \tag{6.4}$$

Using (6.8)

$$M = \frac{1+c}{r+c} MB \tag{6.5}$$

7 The flow government budget constraint

$$PD_t \equiv Q_t - T_t \quad PS_t \equiv T_t - Q_t \tag{7.1}$$

$$TD_t \equiv Q_t + iB_{t-1} - T_t \quad TS_t \equiv T_t - (Q_t + iB_{t-1}) \tag{7.2}$$

$$B_1 = (1+i)B_0 + Q1 - T_1 \Leftrightarrow B_1 - B_0 = iB_0 + PD_1 \Leftrightarrow B_1 - B_0 = TD_1 \tag{7.3}$$

$$B_1 = (1+i)B_0 + Q1 - T_1 \Leftrightarrow B_1 - B_0 = iB_0 - PS_1 \Leftrightarrow B_1 - B_0 = -TS_1 \tag{7.4}$$

(7.3) and (7.4) are all different ways to write the same thing, which can be writed as:

$$B_1 = (1+i)B_0 - PS_1 \tag{7.5}$$

8 The stock government budget constraint

$$B_0 = (1+i)B_{-1} + PD_0 \tag{8.1}$$

Shifting backward by one period

$$B_{-1} = (1+i)B_{-2} + PD_{-1} \tag{8.2}$$

And using (8.2) to replace the term B_{-1} in (8.1)

$$B_0 = (1+i)^2 B_{-2} + (1+i)PD_1 + PD_0 \tag{8.3}$$

If we keep going we get

$$B_0 = (1+i)^N B_{-N} + \sum_{i=0}^{N-1} (1-r)^i PD_{-i} \tag{8.4}$$

9 The debt/GDP ratio

$$B_t = (1+i_{t-1})B_{t-1} - PS_t \tag{9.1}$$

We know that we can write the nominal interest rate as

$$1 + i_{t-1} = 1 + r + \pi^e_{t/t-1} \tag{9.2}$$

Replacing in (9.1) becomes

$$B_t = (1 + r + \pi^e_{t/t-1})B_{t-1} - PS_t \tag{9.3}$$

We know that $Y_t = y_t P_t$. We divide both sides of equation (9.3) by $y_t P_t$

$$\frac{B_t}{y_t P_t} = (1 + r + \pi^e_{t/t-1})\frac{B_{t-1}}{y_t P_t} - \frac{PS_t}{y_t P_t} \tag{9.4}$$

We multiply and divide the right term by $t_{t-1}P_{t-1}$

$$\frac{B_t}{y_t P_t} = (1 + r + \pi^e_{t/t-1})\frac{B_{t-1}}{y_{t-1}P_{t-1}}\frac{y_{t-1}P_{t-1}}{y_t P_t} - \frac{PS_t}{y_t P_t} \tag{9.5}$$

We know that $b_{t-1} = \frac{B_{t-1}}{y_{t-1}P_{t-1}}$, $ps_t = \frac{PS_t}{y_t P_t}$

$$b_t = (1 + r + \pi^e_{t/t-1})b_{t-1}\frac{y_{t-1}P_{t-1}}{y_t P_t} - ps_t \tag{9.6}$$

We define γ as the rate of growth of real GDP: $\gamma \equiv \frac{y_t}{y_{t-1}} - 1$. We also define π_t as the inflation rate: $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$. Therefore, we can write (9.6) as

$$b_t = \frac{(1 + r + \pi^e_{t/t-1})}{(1 + \gamma)(1 + \pi_t)}b_{t-1} - ps_t \tag{9.7}$$

We can simplify this expression using the following approximation

$$(1 + \gamma)(1 + \pi_t) \approx (1 + \gamma + \pi_t) \tag{9.8}$$

Now we can write (9.7) as

$$b_t = \frac{(1 + r + \pi^e_{t/t-1})}{(1 + \gamma + \pi_t)}b_{t-1} - ps_t \tag{9.9}$$

We know that $\log\left[\frac{(1+x)}{(1+z)}\right] = log(1 + x) - log(1 + z) \approx x - z$. If we apply this to $\frac{(1+r+\pi^e_{t/t-1})}{(1+\gamma+\pi_t)}$

$$\log\left[\frac{(1 + r + \pi^e_{t/t-1})}{(1 + \gamma + \pi_t)}\right] = \log(1 + r + \pi^e_{t/t-1}) - \log(1 + \gamma + \pi_t) \tag{9.10}$$

$$\Rightarrow 1 + r + \pi^e_{t/t-1} - 1 - \gamma - \pi_t \Rightarrow r + \pi^e_{t/t-1} - \gamma - \pi_t \tag{9.11}$$

This equation without log would be $1 + r + \pi^e_{t/t-1} - \gamma - \pi_t$. Therefore, we can write (9.9) as

$$b_t = (1 + r - \gamma + \pi^e_{t/t-1} - \pi_t)b_{t-1} - ps_t \tag{9.12}$$

$$\Leftrightarrow b_t - b_{t-1} = (r - \gamma + \pi^e_{t/t-1} - \pi_t)b_{t-1} - ps_t \tag{9.13}$$

10 What happens to the debt/GDP ratio

We define the Steady State as the state in which $b_t = b_{t-1}$

$$b_t - b_{t-1} \Rightarrow b_{ss} = \frac{ps}{r - \gamma} \tag{10.1}$$

Also, we suppose that unexpected inflation is 0 then

$$b_t - b_{t-1} = (r - \gamma)b_{t-1} - ps_t \tag{10.2}$$

10.1 Case 1: $r > \gamma$

If we start above the Steady State

$$b_{t-1} > b_{ss} \Rightarrow b_{t-1} > \frac{ps}{r-\gamma} \tag{10.3}$$

$$b_t - b_{t-1} = (r-\gamma)b_{t-1} - ps \tag{10.4}$$

We can substitute b_{t-1} with b_{ss} as we know that $b_{t-1} > b_{ss}$

$$b_t - b_{t-1} > (r-\gamma)b_{ss} - ps \tag{10.5}$$

$$b_t - b_{t-1} > (r-\gamma)\frac{ps}{r-\gamma} - ps = 0 \tag{10.6}$$

$$b_t - b_{t-1} > 0 \Rightarrow b_t > b_{t-1} \tag{10.7}$$

Hence, if $r > \gamma$ and we start above the Steady State, the debt/GDP ratio will diverge and keep increasing.
If we start below the Steady State

$$b_{t-1} < b_{ss} \Rightarrow b_{t-1} < \frac{ps}{r-\gamma} \tag{10.8}$$

$$b_t - b_{t-1} = (r-\gamma)b_{t-1} - ps \tag{10.9}$$

We can substitute b_{t-1} with b_{ss} as we know that $b_{t-1} < b_{ss}$

$$b_t - b_{t-1} < (r-\gamma)b_{ss} - ps \tag{10.10}$$

$$b_t - b_{t-1} < (r-\gamma)\frac{ps}{r-\gamma} - ps = 0 \tag{10.11}$$

$$b_t - b_{t-1} < 0 \Rightarrow b_t < b_{t-1} \tag{10.12}$$

Hence, if $r > \gamma$ and we start below the Steady State, the debt/GDP ratio will diverge and keep decreasing.

10.2 Case 2: $r < \gamma$

If we start above the Steady State

$$b_{t-1} > b_{ss} \Rightarrow b_{t-1} > \frac{ps}{r-\gamma} \quad (r-\gamma)b_{t-1} < ps \tag{10.13}$$

$$b_t - b_{t-1} = (r-\gamma)b_{t-1} - ps \tag{10.14}$$

We can substitute b_{t-1} with b_{ss} as we know that $b_{t-1} > b_{ss}$

$$b_t - b_{t-1} < (r-\gamma)b_{ss} - ps \tag{10.15}$$

$$b_t - b_{t-1} < (r-\gamma)\frac{ps}{r-\gamma} - ps = 0 \tag{10.16}$$

$$b_t - b_{t-1} < 0 \Rightarrow b_t < b_{t-1} \tag{10.17}$$

Hence, if $r < \gamma$ and we start above the Steady State, the debt/GDP ratio will converge to the Steady State.
If we start below the Steady State

$$b_{t-1} < b_{ss} \Rightarrow b_{t-1} < \frac{ps}{r-\gamma} \quad (r-\gamma)b_{t-1} > ps \tag{10.18}$$

$$b_t - b_{t-1} = (r-\gamma)b_{t-1} - ps \tag{10.19}$$

We can substitute b_{t-1} with b_{ss} as we know that $b_{t-1} < b_{ss}$

$$b_t - b_{t-1} > (r-\gamma)b_{ss} - ps \tag{10.20}$$

$$b_t - b_{t-1} > (r-\gamma)\frac{ps}{r-\gamma} - ps = 0 \tag{10.21}$$

$$b_t - b_{t-1} > 0 \Rightarrow b_t > b_{t-1} \tag{10.22}$$

Hence, if $r < \gamma$ and we start below the Steady State, the debt/GDP ratio will converge to the Steady State.

11 Debt sustainability analysis

Suppose we are in case 1 i.e. $r > \gamma$ (case 2 is not that much a problem as the ratio will converge to the Steady State).

$$b_{t+1} - b_t = (r-\gamma)b_t - ps \tag{11.1}$$

If we want to stabilize the debt/GDP ratio:

$$(r-\gamma)b_t - ps = 0 \tag{11.2}$$

$$\Leftrightarrow ps = (r-\gamma)b_t \tag{11.3}$$

12 Intertemporal budget constraint of the Government

$$B_1 = (1+i)B_0 - PS_1 \tag{12.1}$$

$$B_2 = (1+i)B_1 - PS_2 \tag{12.2}$$

Suppose that the wolrd ends at the end of period 2. The government cannot end with positive debt because this would mean that some creditor would not get repaid at the end of the world: nobody would want to lend to the government hence we can exlude the case $B_2 > 0$. For the same reason is not optimal for the Government to end with negative debt, hence we can exclude the case $B_2 < 0$. We can affirm that at the end of period 2, $B_2 = 0$. From (12.2)

$$0 = (1+i)B_1 - PS_2 \tag{12.3}$$

$$B_1 = \frac{PS_2}{(1+i)} \tag{12.4}$$

Substituting in (12.1)

$$\frac{PS_2}{(1+i)} + PS_1 = (1+i)B_0 \tag{12.5}$$

$$B_0 = \frac{PS_2}{(1+i)^2} + \frac{PS_1}{(1+i)} \tag{12.6}$$

We can rewrite (12.6) as

$$B_0 = \left[\frac{T_1}{(1+i)} + \frac{T_2}{(1+i)^2}\right] - \left[\frac{G_1}{(1+i)} + \frac{G_2}{(1+i)^2}\right] \tag{12.7}$$

13 Intertemporal budget constraint of the private sector

$$A_1 = (1+i)A_0 + Y_1 - T_1 - C_1 \tag{13.1}$$

$$A_2 = (1+i)A_1 + Y_2 - T_2 - C_2 \tag{13.2}$$

Combining (13.1) and (13.2) we obtain

$$A_2 = (1+i)^2 A_0 + (1+i)(Y_1 - T_1 - C_1) + (Y_2 - T_2 - C_2) \tag{13.3}$$

We define private savings as $S_t^{PR} = Y_t - T_t - C_t$ hence we can rewrite (13.3) as

$$A_2 = (1+i)^2 A_0 + (1+i)S_1^{PR} + S_2^{PR} \tag{13.4}$$

By analogy with the case of the government, at the end of period 2, $A_2 = 0$, hence

$$A_0 = -\frac{S_1^{PR}}{(1+1)} - \frac{S_2^{PR}}{(1+1)^2} \tag{13.5}$$

$$A_0 = -\frac{(Y_1 - T_1 - C_1)}{(1+1)} - \frac{(Y_2 - T_2 - C_2)}{(1+1)^2} \tag{13.6}$$

Rearranging (13.6)

$$\frac{C_1}{(1+i)} + \frac{C_2}{(1+i)^2} = A_0 + \frac{Y_1 - T_1}{(1+i)} + \frac{Y_2 - T_2}{(1+i)^2} \tag{13.7}$$

14 Pure tax change in neoclassical approach

We define a pure tax change as a change in taxes which is not accompanied by a change in government spending in either of the two periods. In particulare let's consider the effects of a tax cut in period 1: $\Delta T_1 < 0$.
Assumption: $\Delta G_1 = \Delta G_2 = 0$
From the intertemporal budget constraint of the Government:

$$\Delta B_0 = \Delta\left[\frac{T_1}{(1+i)} + \frac{T_2}{(1+i)^2}\right] - \Delta\left[\frac{G_1}{(1+i)} + \frac{G_2}{(1+i)^2}\right] \tag{14.1}$$

As $\Delta B_0 = 0$ and by assumption the present value of Government spending doesn't change:

$$\frac{\Delta T_1}{(1+i)} + \frac{\Delta T_2}{(1+i)^2} = 0 \tag{14.2}$$

Hence, taxes in period must increase by the same amount (in present value terms) as the tax cut in period 1. From the private sector's budget constraint:

$$\left[\frac{C_1}{(1+i)} + \frac{C_2}{(1+i)^2}\right] = A_0 + \left[\frac{Y_1}{(1+i)} + \frac{Y_2}{(1+i)^2}\right] - \left[\frac{T_1}{(1+i)} + \frac{T_2}{(1+i)^2}\right] \tag{14.3}$$

$$\left[\frac{\Delta C_1}{(1+i)} + \frac{\Delta C_2}{(1+i)^2}\right] = \Delta A_0 + \left[\frac{\Delta Y_1}{(1+i)} + \frac{\Delta Y_2}{(1+i)^2}\right] - \left[\frac{\Delta T_1}{(1+i)} + \frac{\Delta T_2}{(1+i)^2}\right] \tag{14.4}$$

We know that $\Delta A_0 = 0$ and from (14.2) the present value of the tax change is equal to 0.

$$\left[\frac{\Delta C_1}{(1+i)} + \frac{\Delta C_2}{(1+i)^2}\right] = \left[\frac{\Delta Y_1}{(1+i)} + \frac{\Delta Y_2}{(1+i)^2}\right] \tag{14.5}$$

$$IF \left[\frac{\Delta Y_1}{(1+i)} + \frac{\Delta Y_2}{(1+i)^2}\right] = 0 \Rightarrow \left[\frac{\Delta C_1}{(1+i)} + \frac{\Delta C_2}{(1+i)^2}\right] = 0 \tag{14.6}$$

15 Pure Government spending change in neoclassical approach

Assumption: $\Delta G_1 > 0$ and $\Delta G_2 = 0$. We also assume that ouptut doesn't change.
From the intertemporal budget constraint of the government:

$$\Delta B_0 = \left[\frac{\Delta T_1}{(1+i)} + \frac{\Delta T_2}{(1+i)^2}\right] - \left[\frac{\Delta G_1}{(1+i)} + \frac{\Delta G_2}{(1+i)^2}\right] \tag{15.1}$$

We suppose that $\Delta B_0 = 0$ and as we assumed that $\Delta G_2 = 0$ we can rewrite (15.1) as

$$\left[\frac{\Delta T_1}{(1+i)} + \frac{\Delta T_2}{(1+i)^2}\right] = \frac{\Delta G_1}{(1+i)} \tag{15.2}$$

From the intertemporal budget constraint of private sector

$$\left[\frac{\Delta C_1}{(1+i)} + \frac{\Delta C_2}{(1+i)^2}\right] = \Delta A_0 + \left[\frac{\Delta Y_1}{(1+i)} + \frac{\Delta Y_2}{(1+i)^2}\right] - \left[\frac{\Delta T_1}{(1+i)} + \frac{\Delta T_2}{(1+i)^2}\right] \tag{15.3}$$

We know that $\Delta A_0 = 0$, that input doesn't change for assumption and that the present value of the taxes is equal to $\frac{\Delta G_1}{(1+i)}$

$$\left[\frac{\Delta C_1}{(1+i)} + \frac{\Delta C_2}{(1+i)^2}\right] = -\frac{\Delta G_1}{(1+i)} < 0 \tag{15.4}$$

If $i = 0$ and for the notion of consumption smoothing:

$$\Delta C_1 = \Delta C_2 = -0.5\Delta G_1 \tag{15.5}$$

16 Disposable income effect

$$C_1 = \bar{c} + c(Y_1 - T_1) \tag{16.1}$$

$$Y_1 = C_1 + G_1 \tag{16.2}$$

Substituting (16.1) in (16.2)

$$Y_1 = \frac{\bar{c} + G_1 - cT_1}{1-c} \tag{16.3}$$

We define disposable income YD_1 as $Y_1 - T_1$

$$Y_1 - T_1 = \frac{\bar{c} + G_1 - cT_1}{1-c} - T_1 \tag{16.4}$$

$$YD_1 = \frac{\bar{c} + G_1}{1-c} - \frac{cT_1}{1-c} - \frac{(1-c)T_1}{1-c} \tag{16.5}$$

$$YD_1 = \frac{\bar{c} + G_1}{1-c} - \frac{cT_1 - T_1 + cT_1}{1-c} \tag{16.6}$$

$$YD_1 = \frac{\bar{c} + G_1 - T_1}{1-c} \tag{16.7}$$

Substituing in (16.1)

$$C_1 = \bar{c} + c(\frac{\bar{c} + G_1 - T_1}{1-c}) \tag{16.8}$$

$$C_1 = \frac{\bar{c}(1-c)}{1-c} + \frac{c\bar{c}}{1-c} + \frac{c(G_1 - T_1)}{1-c} \tag{16.9}$$

$$C_1 = \frac{\bar{c} + c(G_1 - T_1)}{1-c} \tag{16.10}$$

In the end

$$\Delta YD_1 = -\frac{1}{1-c} \cdot \Delta T_1 \quad \Delta C_1 = -\frac{c}{1-c} \cdot \Delta T_1 \tag{16.11}$$

17 Distortion effect

$$Y = \left(1 - \frac{h^2\tau^2}{3}\right)\bar{Y} \tag{17.1}$$

The first derivative is

$$\frac{\partial Y}{\partial \tau} = \left(-\frac{2}{3} \cdot h^2\tau\right)\bar{Y} < 0 \tag{17.2}$$

Hence a higher tax rate rreduces output.
The second derivative is

$$\frac{\partial^2 Y}{\partial^2 \tau} = \left(-\frac{2}{3} \cdot h^2\right)\bar{Y} < 0 \tag{17.3}$$

Hence the distortionary effect is higher, the higher the tax rate.

18 Lafferian approach to tax cut

We know that $T = \tau Y$ hence:

$$T = \tau\left(1 - \frac{h^2\tau^2}{3}\right)\bar{Y} \tag{18.1}$$

$$T = \left(\tau - \frac{h^2\tau^3}{3}\right)\bar{Y} \tag{18.2}$$

The first derivative is

$$\frac{\partial T}{\partial \tau} = (1 - h^2\tau^2)\bar{Y} \tag{18.3}$$

The second derivative is

$$\frac{\partial^2 T}{\partial^2 \tau} = (-2h^2\tau)\bar{Y} \tag{18.4}$$

If we want to maximize our tax revenus, we must put (18.3) equal to 0

$$(1 - h^2\tau^2)\bar{Y} = 0 \tag{18.5}$$

$$\bar{Y} - h^2\tau^2\bar{Y} = 0 \tag{18.6}$$

$$\bar{Y} = h^2\tau^2\bar{Y} \tag{18.7}$$

$$h^2\tau^2 = 1 \Leftrightarrow \tau^2 = \frac{1}{h^2} \Leftrightarrow \tau = \sqrt{\frac{1}{h^2}} \Leftrightarrow \tau = \frac{1}{h} \tag{18.8}$$

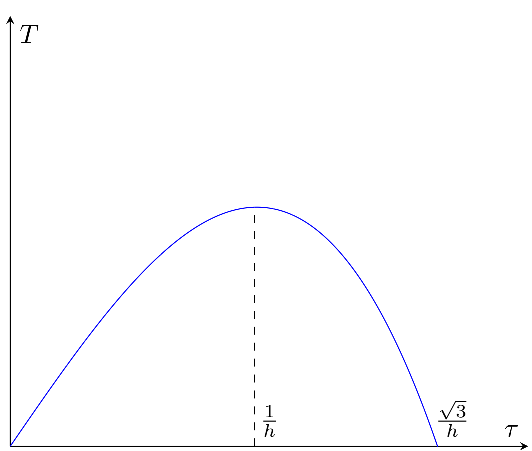

Figure 1: The Laffer curve.

19 Why debt is a problem, burden on future generations

Assumptions: each period correspond to a generation; the world ends at the end of period 2; income is costant at Y; this is a small economy that can borrow at an interest rate $i = 0$; there are not transfers; if there is deficit, the government must issue debt; $B2 = 0$; G is costant.

$$B_1 = G - T_1 \tag{19.1}$$

$$B_2 = B_1 + G - T_2 = 0 \tag{19.2}$$

From (19.2)

$$T_2 = B_1 + G \tag{19.3}$$

Recalling the notion of disposable income $YD_t = Y_t - T_t$; this must be allocated either in consumption or savings. Because there is no reason to save in this model where an individual lives for only one period, consumption is equal to disposable income.

$$C_1 = Y - T_1 \quad C_2 = Y - T_2 \tag{19.4}$$

We now start from the initial equilibrium where $T_1 = T_2 = G$. Both generations have the same consumption $Y - G$. Now suppose the Government reduces taxes on generation 1 by $\Delta T_1 < 0$ and issue debt.

$$\Delta T_1 < 0 \Rightarrow \Delta B_1 = -\Delta T_1 > 0 \tag{19.5}$$

$$\Delta C_1 = \Delta Y - \Delta T_1 = -\Delta T_1 > 0 \tag{19.6}$$

Hence the government issue debt for an amount equal to $-\Delta T_1$ and the consumption of generation 1 increase by the same amount.
Let's see what happen to generation 2. From (19.3)

$$\Delta T_2 = \Delta B_1 + \Delta G = \Delta B_1 = -\Delta T_1 > 0 \tag{19.7}$$

$$\Delta C_2 = \Delta Y - \Delta T_2 = \Delta T_1 < 0 \tag{19.8}$$

Hence for generation 2 taxes increase by an amount equal to the tax reduction in the first period (remember that $i = 0$), while consumption decrease by the same amount.
We define national savings as the sum of government savings and private savings:

$$S = S_{PR} + S_G = (Y - T - C) + (T - G) = Y - C - G \tag{19.9}$$

In the initial equilibrium $T = G$ and $C = Y - T$ hence $S_{PR} = S_G = 0$. What happen to national savings after a tax cut in generation 1?

$$\Delta S_G = \Delta T_1 - \Delta G = \Delta T_1 < 0 \tag{19.10}$$

$$\Delta S_{PR} = \Delta Y_1 - \Delta T_1 - \Delta C_1 = -\Delta T_1 - (-\Delta T_1) = 0 \tag{19.11}$$

$$\Delta S_1 = \Delta S_G + \Delta S_{PR} = \Delta T_1 < 0 \tag{19.12}$$

National savings are lower.

20 Ricardian equivalence

Now suppose that the current generation cares about the future generation. So generation 1 won't consume the tax cut, instead they will bequest that to generation 2. Let Z be the bequests left by generation 1 to their children.

$$C_1 = Y - T_1 - Z \quad C_2 = Y - T_2 + Z \tag{20.1}$$

Generation leave a bequest such that $Z = -\Delta T_1$. Remember that $\Delta T_2 = -\Delta T_1$

$$\Delta C_1 = -\Delta T_1 - Z = 0 \quad \Delta C_2 = -\Delta T_2 + Z = 0 \tag{20.2}$$

Hence

$$\Delta S_{PR} = \Delta Y - \Delta T_1 - \Delta C_1 = -\Delta T_1 > 0 \tag{20.3}$$

$$\Delta S_G = \Delta T_1 - \Delta G = \Delta T_1 < 0 \tag{20.4}$$

$$\Delta S = \Delta S_{PR} + \Delta S_G = -\Delta T_1 + \Delta T_1 = 0 \tag{20.5}$$

As we can see, if the Ricardian equivalence holds, a tax cut won't influence consumption in both generations. Although private savings goes up while public savings goes down, overall national savings won't change.

21 Seignorage

A central bank can buy the debt of a Government. Note that government will pay interests to the central bank for the debt. The central bank gives these interests back to the Government. Hence, it seems that Governments can get something with nothing, it can increase government spending and never increase taxes. We denote the amount of debt held by the central bank as B_t^{cb} and the amount of debt held by the private sector as B_t^p

$$G_t + (1 + i_t)B_{t-1}^p + (1 + i_t)B_{t-1}^{cb} = B_t^p + B_t^{cb} + T_t + i_t B_{t-1}^{cb} \tag{21.1}$$

The left hand side is what the Government must pay in t while the right hand side is what the Government receive in t. Note that we have $i_t B^{cb}_{t-1}$ in both sides because the central bank rebate all his earnings to the Government.

$$G_t + B^p_{t-1} + i_t B^p_{t-1} + B^{cb}_{t-1} + i_t B^{cb}_{t-1} = B^p_t + B^{cb}_t + T_t + i_t B^{cb}_{t-1} \tag{21.2}$$

$$G_t + i_t B^p_{t-1} = B^p_t - B^p_{t-1} + B^{cb}_t - B^{cb}_{t-1} + T_t \tag{21.3}$$

We define $B^{cb}_t - B^{cb}_{t-1} = \Delta MB_t$ as it indicates the change in monetary base.

$$B^p_t - B^p_{t-1} = G_t - T_t + i_t B^p_{t-1} - \Delta MB_t \tag{21.4}$$

$$B^p_t = G_t - T_t + (1 + i_t) B^p_{t-1} - \Delta MB_t \tag{21.5}$$

Rewrite the expression in real terms dividing by P_t

$$\frac{B^p_t}{P_t} = \frac{G_t}{P_t} - \frac{T_t}{P_t} + (1 + i_t)\frac{B^p_{t-1}}{P_t} - \frac{\Delta MB_t}{P_t} \tag{21.6}$$

$$b^p_t = g_t - t_t + (1 + i_t)\frac{B^p_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} - \frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t} \tag{21.7}$$

Recalling that $\frac{P_{t-1}}{P_t} = \frac{1}{1+\pi_t}$

$$b^p_t = g_t - t_t + \frac{(1 + i_t)}{(1 + \pi_t)} b^p_{t-1} - \frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t} \tag{21.8}$$

Rewrite the nominal interest rate as $r + \pi^e_{t+1/t}$

$$b^p_t = g_t - t_t + \frac{(1 + r + \pi^e_{t+1/t})}{(1 + \pi_t)} b^p_{t-1} - \frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t} \tag{21.9}$$

We know that $\log\left[\frac{(1+x)}{(1+z)}\right] = log(1 + x) - log(1 + z) \approx x - z$. If we apply this to $\frac{(1+r+\pi^e_{t+1/t})}{(1+\pi_t)}$ we can linearize (21.9) as

$$b^p_t = g_t - t_t + (1 + r + \pi^e_{t+1} - \pi_t) b^p_{t-1} - \frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t} \tag{21.10}$$

The first term of the expression $\frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t}$, in the long run grows at the same rate as prices i.e is equal to inflation rate.

$$\frac{\Delta MB_t}{MB_t} = \pi \tag{21.11}$$

The second term is the demand for monetary base

$$\frac{MB_t}{P_t} = n_0 - \frac{q-1}{2}(r + \pi^e_{t+1/t}) = q_0 - \frac{q_1}{2}\pi^e_{t+1/t} \tag{21.12}$$

As in the long run there is no difference between expected inflation and inflation

$$\frac{MB_t}{P_t} = q_0 - \frac{q_1}{2}\pi \tag{21.13}$$

From (21.11) and (21.13)

$$\frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t} = q_0\pi - \frac{q_1}{2}\pi^2 \tag{21.14}$$

There is a level of inflation that maximizes the revenues from the monetization of government debt. We take the first derivative of the right hand side of (21.14) and equate it to 0.

$$\frac{\partial \frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t}}{\partial \pi} = q_0 - q_1\pi = 0 \tag{21.15}$$

$$q_0 - q_1\pi = 0 \Rightarrow \pi^* = \frac{q_0}{q_1} \tag{21.16}$$

Hence the maximum seignorage is

$$\left(\frac{\Delta MB_t}{MB_t}\frac{MB_t}{P_t}\right)^{max} = q_0\frac{q_0}{q_1} - \frac{q_1}{2}\left(\frac{q_0}{q_1}\right)^2 = \frac{q_0^2}{q_1} - \frac{q_0^2}{2q_1} = \frac{q_0^2}{2q_1} \tag{21.17}$$

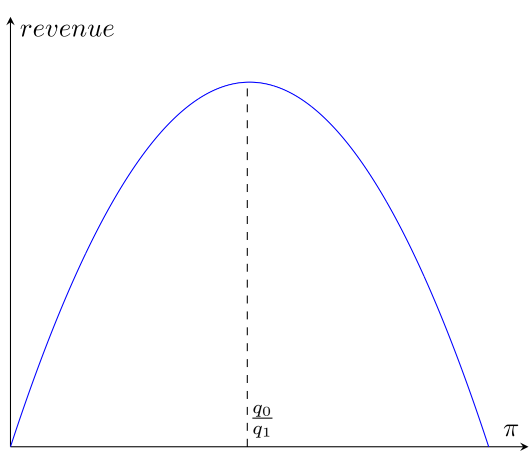

Figure 2:

22 Exchange rates

In this section we will define the real exchange rate. Note that E_B is the nominal exchange rate, P_A and P_B are the price levels respectively in country A and in country B.

$$RER_{P_B} = \frac{E_B P_A}{P_B} \tag{22.1}$$

We can also define the real exchange rate based on unit labour costs.

$$ULC = \frac{Lw(1+t)}{Y} \tag{22.2}$$

Where L is labour input, w the hourly wage, t the rate of social security contribution and labour taxes, Y is output. We define $labour\ productivity = \frac{Y}{L}$

$$ULC = \frac{w(1+t)}{\frac{Y}{L}} \tag{22.3}$$

Hence

$$RER_{ULC_B} = \frac{E_B ULC_A}{ULC_B} \tag{22.4}$$

We can also define real exchange rate based on the consumer prices

$$RER_{CPI_B} = \frac{E_B CPI_A}{CPI_B} \tag{22.5}$$

And, lastly, we can define the real exchange rate as the ratio of non-traded goods to traded goods.

$$RER_{PNT_A} = \frac{PNT_A}{PT_A} \tag{22.6}$$

23 Current account

$$CA = X - M \tag{23.1}$$

$$CA = \Delta NFA \tag{23.2}$$

There is also an important relation between the current account and the government deficit.

$$Y = C + I + G + CA \tag{23.3}$$

And let the savings of the country be $S = Y - C - G$. From 23.3

$$Y - C - G = CA + I \tag{23.4}$$

Therefore

$$S = CA + I \tag{23.5}$$

i.e. total savings can be used to increase to buy foreign assets (CA) or to increase the domestic capital stock (I). We can re-write (23.5) as

$$S_g + S_p = CA + I \tag{23.6}$$

$$S_p = -S_g + CA + I \tag{23.7}$$

i.e. private savings can be used to finance government dis-savings (government deficit), to finance the accumulation of physical capital, to finance the accumulation of net foreign assets (current account surplus). Recalling (23.2), in a fixed exchange rate the central bank holds foreign exchange reserves, to peg his currency to the foreign currency, hence

$$CA = \Delta NFA_p + \Delta NFA_{cb} \tag{23.8}$$

$$\Delta NFA_{cb} = CA - \Delta NFA_p \tag{23.9}$$

i.e. $change\ in\ CB\ reserves = CA + net\ private\ capital\ inflows$

24 Uncovered interest parity condition

$$1 + i_A = (1 + i_B)\frac{E_B}{E_B^e} \tag{24.1}$$

$$i_A = \frac{E_B}{E_B^e} + i_B\frac{E_B}{E_B^e} - 1 \tag{24.2}$$

$$i_A = i_B + \left(\frac{E_B}{E_B^e} - 1\right) \tag{24.3}$$

25 Inflation expectations and fixed exchange rate

Suppose that inflation in country A is $\pi_A > 0$ while in country B is $\pi_B = 0$. Assume that the real interest rate is constant at r. From the Fisher equation

$$i_A = r + \pi_A \tag{25.1}$$

$$i_B = r \tag{25.2}$$

The purchasing power parity theory says that in the long run the real exchange rate is constant

$$\frac{EP_A}{P_B} = RER^* = constant \tag{25.3}$$

$$\frac{dE}{E} + \frac{dP_A}{P_A} - \frac{dP_B}{P_B} = 0 \tag{25.4}$$

Since $\pi_B = 0 \Rightarrow fracdP_BP_B = 0$

$$\frac{dE}{E} = -\frac{dP_A}{P_A} \tag{25.5}$$

Suppose now A fixes the exchange rate with B. Assume that the exchange rate is fixed credibly. Then, expected devaluation is 0 and from the interest parity condition

$$i_A = i_B \tag{25.6}$$

26 Unit labour cost growth

$$ULC = \frac{w(1+t)}{\frac{Y}{L}} \tag{26.1}$$

The rate of change of unit labour cost is

$$\frac{\Delta ULC}{ULC} = \frac{\Delta[w(1+t)]}{w(1+t)} - \frac{\Delta\frac{Y}{L}}{\frac{Y}{L}} \tag{26.2}$$

where the term $\frac{\Delta[w(1+t)]}{w(1+t)}$ is the growth of hourly labour compensation, while the term $\frac{\Delta\frac{Y}{L}}{\frac{Y}{L}}$ is labour productivity growth.