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MICROECONOMICS I PARTIAL

1° BIG

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Microeconomics – first partial

key words

trade-offs

optimization process

demand curve

supply curve

equilibrium price

market

substitutes

complements

market demand curve

market supply curve

equilibrium price

excess of supply

excess of demand

elasticity of demand

elasticity of supply

formulas

$$Q\ demanded = A - BP$$

$$P = \frac{A}{B} - \frac{Q\ demanded}{B}$$

$$Q\ supplied = BP - A$$

$$P = \frac{Q\ supplied}{B} + \frac{A}{B}$$

$$E_Y^X = \frac{\%\ change\ in\ X}{\%\ change\ in\ Y}$$

$$\%\ \Delta X = \frac{100(Xn - Xo)}{Xo}$$

$$\%\ \Delta Y = \frac{100(Yn - Yo)}{Yo}$$

Microeconomics is the **study** of how **agents** (producers and consumers) in a society **use** their **limited resources** to **produce, exchange and consume goods and services**. Since consumers and producers have a limited budget and a limited productivity, they **must make trade-offs** to understand what's their best option; to make trade-off we use the **optimization process**, constructing a **demand and a supply curve** and finding the **equilibrium price.** Prices are determined through the actual or potential interactions between buyers and sellers. A **market** is a **collection of buyers and sellers** who, through these interactions, **determine the price of *highly interchangeable* products**. A market is associated with a single group of **closely related products** that are offered for sale within **particular geographic boundaries** (markets can be *competitive, monopolies* or *oligopolies).*

Demand depends on the **price of the good** (variable), population growth, price of **substitutes** (an increase in the price of competitor's product causes buyers to demand more of the other substitutive good), price of **complements** (an increase in the price of the related product causes buyers to demand less of the complementary good), consumer tastes and **income** (for normal goods when income increases the demand increases; for inferior goods when income increases the demand decreases). The market demand curve shows **how much of the good consumers want to buy at each possible price** (*holding fixed all other factors that affect the demand*). The curve is downward sloping (slope < 0), so for higher prices buyers will demand less of a product. A **change in the price of the product** causes a **movement *along* the demand curve**, a **change in a fixed factor** causes the **entire demand curve to *shift.***

Demand has a linear function: $Q\ demanded = A - BP$ (where A and B are positive constants)

To plot a demand curve we use the inverse demand function: $P = \frac{A}{B} - \frac{Q\ demanded}{B}$

Supply depends on the **price of the good** (variable), prices of inputs (labor/capital), taxation and subsidies, technology, availability of raw materials.
The *market supply curve* shows **how much of the product producers want to sell at each possible price** *holding fixed all other factors that affect the supply*. The curve is upward sloping (slope > 0), because selling is less attractive when the price is lower. A **change in the price of the product** causes a **movement *along* the supply curve**, a **change in a fixed factor** causes the **entire supply curve to *shift.***

Supply has a linear function: $Q\ supplied = BP - A$ (where A and B are positive constants)

To plot a supply curve we use the inverse supply function: $P = \frac{Q\ supplied}{B} + \frac{A}{B}$

The **equilibrium price** is the **price** at which the **amounts supplied** and **demanded** are **equal**, to compute it we put $Qdemanded = Qsupplied$ and solve the equation.

If there is an **excess of supply**, the **sellers lower their prices** to restore the equilibrium of the market, since when P↓ then Q^D↑.
If there is an **excess of demand,** the **buyers increase their prices** to restore the equilibrium of the market, since when P↑ then Q^D↓.
The **ultimate effect on equilibrium is the combination of the separate effects of changes in demand and supply**; we can determine the effect on either price or quantity, but not both, because the net effect depends on the relative size of the change.
The **elasticity of demand (or supply)** is the **ratio of the percentage change in Q^D or Q^S to the percentage change in price** $E_Y^X = \frac{\%\ change\ in\ X}{\%\ change\ in\ Y}$ (where **% change in X =** $\frac{100(Xn - Xo)}{Xo}$ and **% change in Y =** $\frac{100(Yn - Yo)}{Yo}$).
If E_Y^X = n then X increases n% for each 1% increase in Y (if n > 0) and decreases n% for each 1% decrease in Y (if n > 0)

Microeconomics – first partial

key words

elasticity of demand to price

total expenditure

income elasticity of demand

cross price elasticity of demand

elasticity of supply

market demand curve

individual demand curve

formulas

$$E_P^{Qd} = \frac{\% \Delta Qd}{\% \Delta P} = \frac{\delta Qd}{\delta P} \cdot \frac{Po}{Qo}$$

$$E_P^{Qd} = \frac{\delta Qd}{\delta P} \cdot \frac{Po}{Qo} = -\text{b} \cdot \frac{Po}{Qo}$$

$$TE = P \cdot Q$$

$$\%\Delta TE = \%\Delta P + \%\Delta Q$$

$$E_M^{Qd} = \frac{\% \Delta Qd}{\% \Delta M} = \frac{\delta Qd}{\delta M} \cdot \frac{Mo}{Qo}$$

$$E_M^{Qd} = \frac{\delta Qd}{\delta M} \cdot \frac{Mo}{Qo} = \pm\, \text{c} \cdot \frac{Mo}{Qo}$$

$$E_{Py}^{Q_X^D} = \frac{\partial Q_X^D}{\partial P_Y} \cdot \frac{Po^Y}{Qo^X} = \pm\, \text{d} \cdot \frac{Po^Y}{Qo^X}$$

$$E_P^{Q^S} = \frac{\partial Q^S}{\partial P} \cdot \frac{Po}{Qo^S} = b \cdot \frac{Po}{Qo^S}$$

The elasticity of demand to price measures by how much the Q^D changes when price changes by a small amount $E_P^{Qd} = \frac{\% \Delta Qd}{\% \Delta P} = \frac{\partial Qd}{\partial P} \cdot \frac{Po}{Qo}$; we expect $E_P^{Qd} < \mathbf{0}$.

$|E_P^{Qd}| > 1$ elastic demand; $|E_P^{Qd}| < 1$ inelastic demand; $|E_P^{Qd}| = 1$ unit elastic demand

$|E_P^{Qd}| = \infty$ perfectly elastic demand; $|E_P^{Qd}| = 0$ perfectly inelastic demand.

Elasticity changes also **on the same <u>linear</u> demand curve:** $E_P^{Qd} = -\text{b} \cdot \frac{Po}{Qo}$

Total expenditure = **how much consumers spend on a** given **good when price varies**

$TE = P \cdot Q$; $\%\Delta TE = \%\Delta P + \%\Delta Q$

	elastic demand	inelastic demand	unitary elastic demand
price increases	TE decreases	TE increases	TE constant
price decreases	TE increases	TE decreases	TE constant

(total expenditure is largest at a price for which the elasticity equals -1)

Income elasticity of demand: measures by how much the quantity demanded changes when income changes by a small amount: $E_M^{Qd} = \frac{\% \Delta Qd}{\% \Delta M} = \frac{\partial Qd}{\partial M} \cdot \frac{Mo}{Qo}$

<u>if $E_M^{Qd} > 0$ the good is a normal good, if $E_M^{Qd} < 0$ the good is an inferior good</u>.

$$E_M^{Qd} = \frac{\partial Qd}{\partial M} \cdot \frac{Mo}{Qo} = \pm\, \text{c} \cdot \frac{Mo}{Qo}$$

Cross price elasticity of demand: measures by how much the quantity demanded of good X changes when the price of good Y changes by a small amount

$$E_{Py}^{Q_X^D} = \frac{\%\Delta Q_X^D}{\%\Delta P_Y} = \frac{\partial Q_X^D}{\partial P_Y} \cdot \frac{Po^Y}{Qo^X} = \pm\, \text{d} \cdot \frac{Po^Y}{Qo^X}$$

<u>if</u> $E_{Py}^{Q_X^D} > 0$ <u>we have substitute goods,</u>

<u>if</u> $E_{Py}^{Q_X^D} < 0$ <u>we have complement goods,</u>

<u>if</u> $E_{Py}^{Q_X^D} = 0$ <u>the goods aren't related</u>.

Elasticity of supply: measures by how much the quantity supplied of good X changes when the price of X changes by a small amount

$E_P^{Q^S} = \frac{\%\Delta Q^S}{\%\Delta P} = \frac{\partial Q^S}{\partial P} \cdot \frac{Po}{Qo^S} = b \cdot \frac{Po}{Qo^S}$; we expect $E_P^{Q^S} > 0$

$E_P^{Q^S} > 1$ elastic supply; $E_P^{Q^S} < 1$ inelastic supply; $E_P^{Q^S} = 1$ unit elastic supply

$E_P^{Q^S} = \infty$ perfectly elastic supply; $E_P^{Q^S} = 0$ perfectly inelastic supply

Market demand curve = sum of the individual demand curves

An **individual demand curve** tells how many units of good X an individual is willing to buy for every possible price of the good, holding fixed other factors influencing the demand.

Microeconomics – first partial

key words

consumer problem

consumer preferences

budget set

ranking principle

choice principle

indifference curve

bad

rate of substitution

MRS

utility function

formulas

$$MRS = \left|\frac{\partial IC_A}{\partial x}\right|$$

$$MU_X = \frac{\partial U(X,Y)}{\partial X}$$

$$MU_Y = \frac{\partial U(X,Y)}{\partial Y}$$

$$MRS = \frac{MU_X}{MU_Y}$$

To plot an individual demand curve we have to solve the **consumer problem** (how to allocate limited funds to maximize consumer's well being):

1) describe the **consumer preferences** (without considering price or income)
2) **budget constraint** (the alternatives the consumer can afford given prices and income)
3) by **putting together the consumer preferences and the budget set** we get the alternative in the budget set with the highest number associated (**CHOICE OF THE CONSUMER**) → construct one point on every demand curve
4) **change P_X** (holding fixed all other factors) and repeat step 2&3 → do it for every Px from 0 to + ∞.

A consumer chooses among consumption bundles the one that maximizes his utility.

Preferences must follow 2 rationality principles:

- **Ranking principle:** the individual is *able to to express a preference on every bundle* → if this principle is valid then we say that the preferences are **complete** and **transitive** (it means that if A>B and B>C, then A>C);
- **Choice principle:** the individual chooses the bundle that he ranked as his preferred one.

We also assume the individual is gonna follow the "**more is better principle**": given bundle A = (x , y) and B = (x , y), *if bundle B contains more units of x* <u>*and*</u> *y compared to A then the individual will choose bundle B.*

An **indifference curve (i.c.)** is the set of all the consumption bundles towards which the consumer is indifferent; a **family of indifference curves** is the set of all the indifference curves that represents one individual's preferences.
Indifference curves: 1) can't have positive slope; 2) are thin; 3) can't cross each other.

Three different graphs for standard preferences (convex curves), complement goods (L-shaped preferences) or substitute goods (downward sloping lines).

A **bad** is a good such that when the individual consumes it his well-being decreases.

The **rate of substitution between X and Y** $\left(-\frac{\Delta Y}{\Delta X}\right)$ measures by how much the consumer is willing to reduce Y to get extra units of X, remaining on the same i.c.
The **marginal rate of substitution between X and Y (MRS)** measures *by how much the consumer is willing to reduce Y in order to increase X by a small amount in order to remain on the same indifference curve.*

$MRS = \left|\frac{\partial IC_A}{\partial x}\right|$ with A being a bundle on an indifference curve

(the steeper the indifference curve the stronger preference for X over Y)

The utility function associates to every bundle a utility level U(A) based on the preferences of the consumer.
Every point of the utility function represents an indifference curve, the function is increasing because of the **more is better** principle.
The marginal utility of X (MU_X) measures by how much U (X,Y) changes when X changes by a small amount, keeping Y fixed (the same is valid for Y).

$MU_X = \frac{\partial U(X,Y)}{\partial X}^*$; $MRS = \frac{MU_X}{MU_Y}$

Microeconomics – first partial

key words

Cobb-Douglas utility

utility function for perfect complements

utility function for perfect substitutes

budget line

formulas

$U(X,Y) = X^a + Y^b$

$U(X,Y) = \min\left\{\frac{X}{b}, \frac{Y}{a}\right\}$

$U(X,Y) = aX + bY$

$Y = \frac{M}{Py} - \frac{Px}{Py} \cdot X$

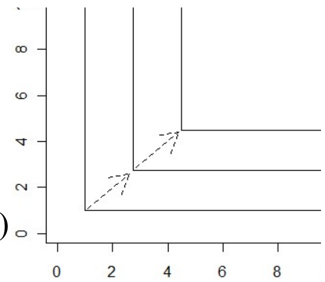
Standard preferences (convex i.c.) have a **COBB-DOUGLAS utility function**:

$U(X, Y) = X^a + Y^b$ (where a, b > 0 and are individual specific preferences)

$MU_X = a X^{a-1} Y^b$; $MU_Y = b X^a Y^{b-1}$; $MRS = \frac{MU_X}{MU_Y} = \frac{a}{b} \cdot \frac{Y}{X}$

Utility function for perfect complements (X and Y are consumed in fixed proportions)

$U(X,Y) = \min\left\{\frac{X}{b}, \frac{Y}{a}\right\}$ (graphically it is L-shaped curve)



to find the corner: $\frac{X}{b} = \frac{Y}{a} \rightarrow Y = \frac{a\,X}{b}$;

MRS = 0 (on the horizontal part);

MRS = ∞ (on the vertical part; in the corner it's *undefined*)

Utility function for perfect substitutes (goods a consumer is willing to substitute at a fixed rate): $U(X, Y) = aX + bY$ (a, b > 0 indicate relative preferences over X and Y); $MRS = \frac{MU_X}{MU_Y} = \frac{a}{b}$

Affordable bundles: **(X, Y) is affordable if $P_X X + P_Y Y \leq M$** (where M is the income).

A budget line separates what's affordable for the consumer from what is not.

Budget line: $P_X X + P_Y Y = M \rightarrow Y = \frac{M}{Py} - \frac{Px}{Py} \cdot X$; slope = $-\frac{Px}{Py}$ (– the price ratio)

$P_X\uparrow$ = line rotates (Y-intercept constant); $P_Y\uparrow$ = line rotates (X-intercept constant); $M\uparrow$ = budget line shifts right

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Consumer's choice given price and income

- For standard preferences (or Cobb-Douglas) utility is maximized in the **intersection** between the family of **indifference curves** and the **budget line** represents the bundle whose ***utility is maximized.***

$$\begin{cases} \frac{P_X}{P_Y} = MRS \rightarrow \frac{P_X}{P_Y} = \frac{a}{b} \cdot \frac{Y}{X} \\ P_X X + P_Y Y = M \end{cases}$$

- For Complement goods (L-shaped indifference curves) to find the bundle whose utility is maximized:

$$\begin{cases} ax = by \rightarrow \frac{x}{b} = \frac{y}{a} \\ P_X X + P_Y Y = M \end{cases}$$

- For substitute goods, to find the bundle whose utility is maximized:

if $MRS > \frac{P_X}{P_Y}$

$$\begin{cases} y = 0 \\ x > 0 \\ P_X X + P_Y Y = M \end{cases}$$

if $MRS < \frac{P_X}{P_Y}$

$$\begin{cases} x = 0 \\ y > 0 \\ P_X X + P_Y Y = M \end{cases}$$

if $MRS = \frac{P_X}{P_Y}$

every bundle that satisfies $P_X X + P_Y Y = M$

Microeconomics – first partial

key words

price-consumption curve

individual's demand curve

Giffen Goods

income effect

substitution effect

Engel curve

labor supply

demand for leisure

formulas

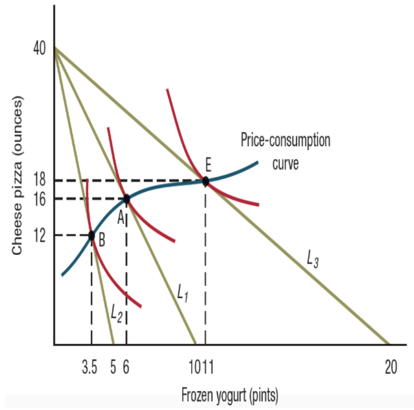
L = T – N

$P_F \cdot F = E + WL$

The **price-consumption** curve shows *how the optimal consumption bundle changes as the price of one good changes*, holding everything else fixed. Price-consumption curve includes all information needed to plot an **individual's demand curve**:

$Q^D = D(price, other\ factors)$ *(It describes the relationship between the price of a good and the amount a particular consumer purchases, holding everything else fixed)*

Starting from the price (P_X or P_Y), a consumption curve finds the demand curve for that good.



<u>*Law of the demand*</u>: if the price of a good X increases ($P_X\uparrow$), Q^D for good X decreases. **Giffen Goods** are an exception: as price increases, demand for these goods <u>increases</u>.

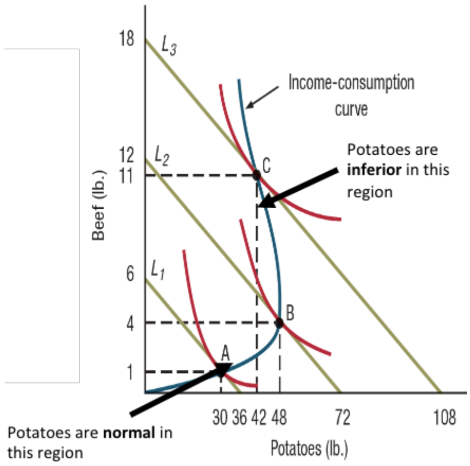
The effect of the change in the price of good 1 on the demand of good 2 will depend on whether they are substitutes or complements:

- **SUBSTITUTES**: decrease in P_1 → leftward shift in the demand for P_2
- **COMPLEMENTS**: decrease in P_1 → rightward shift in the demand for P_2

An **income effect** is the change in the consumption of a good that results from a change in income.

The **income-consumption curve** shows *how the best affordable bundle changes as income changes*, holding everything else fixed.

If a good is **normal** an <u>increase in income raises the amount consumed</u>, if a good is **inferior** an <u>increase in income reduces the amount consumed.</u>



The **Engel curve** for a good describes the **relationship between income and the amount consumed**, holding everything else fixed.

→ Engel curve measures income on the Y-axis and amount consumed on the X-axis.

→ Engel curve slopes upward for normal goods and downward for inferior goods

The demand curve shows the relationship between the price of a good and the amount purchased, including *income* → when income changes the demand curve shifts.

Labor supply refers to the <u>sale of a consumer's time and efforts to an employer.</u>

To understand the supply of labor we study the **demand for leisure,** because people regard <u>hours of work as a "bad"</u> and <u>hours of leisure as a normal good</u>.

Indeed, **wage** = the price of hours of leisure.

People make a traded-off deciding their ***labor-leisure*** function**:**

Example with 2 goods (F=food; N=leisure time):	L(labor) = T(time) – N Preferences are standard and goods are normal.
Now we find the budget constraint:	M(income)= initial wealth (E) + L (labor income) P_F = price of food; **w**(wage) = price of leisure time budget line: $p_F \cdot F = E + WL$ → $F = \frac{E}{P_F} + \frac{WT}{P_F} - \frac{WN}{P_F}$

Microeconomics – first partial

key words

leisure demand curve

labor supply curve

demand curve for leisure time

principal

interest

interest rate

present discounted value

formulas

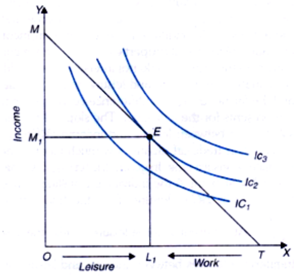
interest rate: $\frac{interest}{principal} \cdot 100\%$

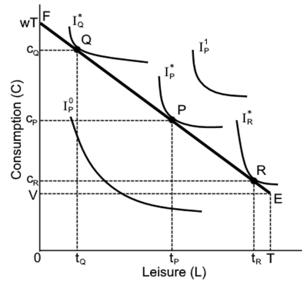
pres. discounted value: $\frac{M}{I+i}$

in future terms M_0 (i + I)

choice:

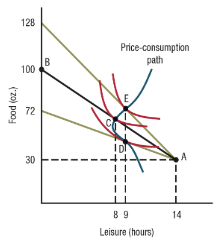
standard	$MRS = \frac{W}{P_F}$ $P_F F = E + W(T - N)$	(N* , F*) L* = T – N* N* > T not a solution
complements	$\frac{N}{b} = \frac{F}{a}$ $P_F F = E + W(T - N)$	(N* , F*) L* = T – N* N* > T not a solution
substitutes	compare MRS with $\frac{W}{P_F}$ $P_F F = E + W(T - N)$	(N* , F*) L* = T or L* = 0 Or L* = T – N* N* > T not a solution

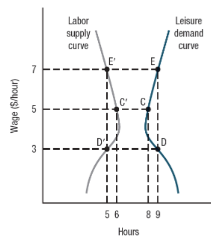




How does a change in wage affect a consumer's budget line?
We look at $pC = w(T - N)$:

- Y-intercept increases as w increases;
- X-intercept doesn't depend on w;
- The budget line becomes **<u>steeper</u>** with an ↑ in w.





Points of tangency between indifference curves and rotating budget lines form a price-consumption path;

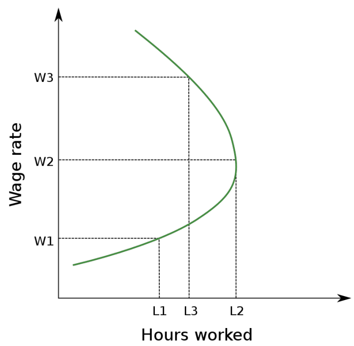
this leads to the **leisure demand curve** and **labor supply curve** (mirror image)

The ***demand curve for leisure time*** tells how many hours of leisure a worker decides to "consume" for every possible wage, holding fixed p_F and E.

To compute the labor supply curve we start from the demand curve for leisure, and we find L = T – N.

In our example demand for leisure is upward sloping for high enough wages. Increase in wage reduces the supply of labor for some range of wages. As a consequence some people may have backward bending labor supply curves.

An increase in wage can make people work less but never make people stop working, a decrease in wage can make people work more, but those who weren't working will continue not to work.



Consumers can be **buyers** (borrow money) or **savers** (save or lend money).

Principal: the capital that is borrowed or saved.
Interest: the price of the principal over a certain period

Interest rate (R): $\frac{interest}{principal} \cdot 100\ \%$

Present discounted value (PDV):

- $\frac{M}{1+i}$ future money discounted at today's value;
- in future terms M_0 (i + 1).

Microeconomics – first partial

key words

borrower

saver

intertemporal budget constraint

present discounted value of budget constraint

formulas

$P_0C_0\,(I+i)+P_1C_1=M_0\,(I+i)+M_1$

$P_0C_0+\frac{P1C1}{I+i}=M_0+\frac{M1}{I+i}$

Solving the consumer problem:
2 goods: C_0 today consumption good, price P_0, C_1 tomorrow consumption good, price P_1
2 periods: t_0 today, t_1 tomorrow
consumers can borrow/lend money between t_0 and t_1 at the interest i(M,R)
M_0 income earned today, M_1 income earned tomorrow from working.
If $\mathbf{P_0C_0 < M_0}$ consumer is a **saver**; if $\mathbf{P_0C_0 > M_0}$ consumer is a **borrower**.

1) PREFERENCES:

Cobb Douglas: $U(C_0\,,C_1) = C_0^a, C_1^b$

Complements: $U(C_0\,,C_1) = \min\{\frac{C_0}{b},\frac{C_1}{a}\}$

Substitutes: $U(C_0\,,C_1) = aC_0 + bC_1$

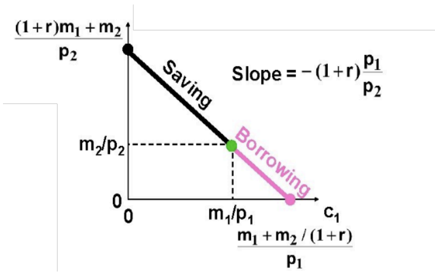
2) INTERTEMPORAL BUDGET CONSTRAINT:

a) $P_0C_0 < M_0$: saver → next year $(M_0 - P_0C_0)\,(I+i) + M_1 = P_1C_1$
→ $P_0C_0\,(1+i) + P_1C_1 = M_0\,(1+i) + M_1$ → budget constraint at tomorrow's value
budget constraint in today's terms: $P_0C_0+\frac{P1C1}{1+i} = M_0+\frac{M1}{1+i}$; present discounted value of budget constraint

b) $P_0C_0 > M_0$: borrower → next year $M_1 - (P_0C_0 - M_0)\,(1+i) = P_1C_1$
→ $P_0C_0\,(1+i) + P_1C_1 = M_0\,(1+i) + M_1$ → budget constraint at tomorrow's value
budget constraint in today's terms: $P_0C_0+\frac{P1C1}{1+i} = M_0+\frac{M1}{1+i}$; present discounted value of budget constraint

slope $=\frac{\partial C1}{\partial C0}$; solve for C_1: $C_1 = \frac{1+i}{P1}\cdot M_0 + \frac{M1}{P1} - P_0\,\frac{1+i}{P1}\cdot C_0 \; - P_0\,\frac{1+i}{P1} = \text{slope}$

Endowment point (•) is where the consumer spends the amount of money owned. If interest rate increases the line rotates to the right, if interest rate decreases the line rotates to the left , budget line is not affected by R and **rotates around** •.



3) CHOICE:

convex curves $U(C_0\,,C_1) = C_0^a\,C_1^b$	substitute goods $U(C_0\,,C_1) = a\,C_0 + b\,C_1$	complement goods $U(C_0\,,C_1) = \min\{\frac{C_0}{b},\frac{C_1}{a}\}$
$\begin{cases} MRS = \frac{P_0\,(1+i)}{P_1} \\ P_0C_0+\frac{P1C1}{1+i} = M_0+\frac{M1}{1+i}\end{cases}$ (C_0*, C_1*) ; if $C_0* < \frac{M_0}{P_0}$ saver if $C_0* > \frac{M_0}{P_0}$ borrower	compare MRS and $\frac{P_0\,(1+i)}{P_1}$ $MRS < \frac{P_0\,(1+i)}{P_1}$ → $C_0* = 0$ $C_1* = \frac{M_0\,(1+i)+M_1}{P_1}$; $MRS > \frac{P_0\,(1+i)}{P_1}$ → $C_1* = 0$ $C_0* = \frac{M_0+\frac{M_1}{1+i}}{P_0}$; $MRS = \frac{P_0\,(1+i)}{P_1}$ every (C_0*, C_1*) that satisfies budget constraint	$\begin{cases} \frac{C_0}{b} = \frac{C_1}{a} \\ P_0C_0+\frac{P1C1}{1+i} = M_0+\frac{M1}{1+i}\end{cases}$

Microeconomics – first partial

key words

market supply curve

input

output

production function

variable input

fixed input

short run

long run

average product of labor

average product of capital

marginal product of labor

marginal product of capital

free disposal

productive inputs principle

law of diminishing returns

formulas

$\pi(Q) = TR - TC$

$Q = a L$

$Q = LK$

$AP_L = \frac{f(L,K)}{L}$

$AP_K = \frac{f(L,K)}{K}$

$MP_L = \frac{\partial f(L,K)}{\partial L}$

$MP_K = \frac{\partial f(L,K)}{\partial K}$

Market supply curve: tells how many units of the good are sold for every possible price, holding fixed the other factors influencing supply. It's the sum of individual supply curves, we expect firms to choose Q for a given price in order to maximize profits profit function: $\pi(Q)$ = total revenues (TR) – total costs (TC) → $\pi(Q) = P*Q - TC$

<u>To construct the function:</u>

1) Understand how many inputs we need to produce Q; production function → associates to every input combination the output they can produce;
2) we construct TC(Q) as the cost associated to the cheapest input combination to produce Q
3) we find the quantity that maximizes $\pi(Q)$ for a given price p
4) we find the quantity that maximizes $\pi(Q)$ for every price

inputs : factors a firm needs to produce a good; they are labor (L), capital (K)
output : the good produced by a firm (indicate with letter Q)
production function : Q = F (inputs) ; it associates to every input combination the output they can produce, in this course Q = f (L , K)
variable input : an input we can modify (in its quantity) in the time period considered, the quantity used depends on how many units a firm wants to produce (i.e. **labor**)
fixed input : an input that can't be modified in the time period considered, doesn't depend on how much a firm wants to produce (i.e. **capital**)

Production in the **short run**: **some** inputs are **fixed**
Production in the **long run**: **<u>all</u>** inputs become **variable**.

Production function in the short run: Q = a L   (K = a; fixed)
Production function in the long run: Q=LK (∞ combinations of L and K to produce Q_1)

Average product of labor measures how much one unit of labor contributes to production *on average*; to compute it we fix the capital level, $AP_L = \frac{f(L,K)}{L}$ (K fixed).
Average product of capital measures how much one unit of capital contributes to production *on average*; to compute it we fix the labor level, $AP_K = \frac{f(L,K)}{K}$ (L fixed).

Marginal product of labor measures by how much Q increases when labor increases by a small amount, holding fixed capital, $MP_L = \frac{\partial f(L,K)}{\partial L}$ partial derivative of f(L , K) with respect to L.
Marginal product of capital measures by how much Q increases when capital increases by a small amount, holding fixed labor, $MP_K = \frac{\partial f(L,K)}{\partial K}$ partial derivative of f(L , K) with respect to K.

Assumptions on the production function Q = f(L , K):

1) ***free disposal***: a firm can freely dispose of every input (if an input may decrease production the firm doesn't use it → production function is <u>never</u> decreasing)
2) ***productive inputs principle***: if we increase both labor and capital production must increase
3) ***law of diminishing returns***: MP_L and MP_K are decreasing (= productivity of each additional unit of L / unit of K is smaller than the productivity of previous units)

Microeconomics – first partial

key words

isoquant

marginal rate of technical substitution

formulas

$MRTS = \frac{MP_L}{MP_K} = \frac{\partial f(L,K)}{\partial L} / \frac{\partial f(L,K)}{\partial K}$

Isoquant: set of all (L, K) that give the same production level Q

Family of isoquants: set of all isoquants describing the same *f* (L , K).

An isoquant divides the plan in two: production is higher in the upper part; production is lower in the lower part; Q increases as the isoquant is further from the origin.

Rate of substitution between L and K tells by how much a firm must decrease K when it increases L in order to produce the Q —> **MARGINAL RATE OF TECHNICAL SUBSTITUTION (MRTS) tells by how much K must decrease if the firm increases L by a small amount, in order to keep producing the same amount of Q.**

$MRTS = \frac{MP_L}{MP_K} = \frac{\partial f(L,K)}{\partial L} / \frac{\partial f(L,K)}{\partial K}$

Higher MRTS = higher MP_L compared to MP_K (decrease a lot K when you increase by a little L)

production functions:

1) CONVEX ISOQUANT: $Q = A L^a K^b$
   free disposal is valid
   productive inputs principle is valid
   MP_L and MP_K decreasing is valid only if a < 1 and b < 1
2) PERFECT SUBSTITUTES (inputs can be substituted at a fixed rate) Q = aL + bK; $MRTS = \frac{a}{b}$
   free disposal is valid
   productive inputs principle is valid
   MP_L and MP_K decreasing **is not valid** (they're constant: $MP_L = a$, $MP_K = b$)
3) PERFECT COMPLEMENTS (use inputs in fixed proportions) Q = min{aL , bK}
   MRTS = ∞ vertical part, = 0 horizontal part, not defined in the corner

For every combination (L , K) the cost of the inputs is WL + rK

1) compute the cost of every (L , K) on the isoquant Q (fixed)
2) select the cheapest (L* , K*) → WL* + rK* is the total cost of producing Q
3) redo step 1 and 2 for every Q → total cost function ITC (Q)

Construct total cost function TC (Q) with 1 variable input (L), K is fixed (short run)
→ in the **short run we can't minimize the cost of production because K is fixed**

to find TC (Q)

1) Q = F(L)
2) L = *f* (Q) $(f = F^{-1})$
3) WL + rK = TC (Q)
4) construct TC (Q) for every Q

In the **long run we can choose every (L, K) on the isoquant cause both L and K are variable** → we want the cheapest one.

How to associate a cost to every (L , K)? → cost of a: $wL_a + rK_a$; cost of b: $wL_b + rK_b$

(L , K) such that their cost is TC (fixed).

TC = wL + rK → isocost: set of all (L , K) that cost TC

Family of isocosts: each with a different TC but with the same W and R slope (slope: –w/r); the further from the origin the more expensive.

The tangency point betweeen isoquant and isocost is the input combination that minimizes the cost of producing Q_1.

Microeconomics – first partial

key words

constant return to scale

increasing return to scale

decreasing return to scale

variable cost

fixed cost

opportunity cost

average cost

marginal cost

economies of scale

diseconomies of scale

formulas

total cost (TC) = VC + FC

$$AC = \frac{C(Q)}{Q}$$

$$MC = \frac{\Delta C}{\Delta Q} = \frac{\partial C(Q)}{\partial Q}$$

$AVC = \frac{VC(Q)}{Q}$

$AFC = \frac{FC(Q)}{Q}$

AC = AVC + AFC

mathematically:

convex isoquants $Q = A L^a K^b$	substitute inputs $Q = aL + bK$	complement inputs $Q = \min\{aL, bK\}$
$\begin{cases} MRTS = \frac{W}{r} \\ F(L, K) = Q_1 \end{cases}$	if $MRTS > \frac{W}{r}$ $K = 0$, $L = \frac{Q_1}{a}$ $TC(Q_1) = w\frac{Q_1}{a}$; if $MRTS < \frac{W}{r}$ $L = 0$, $L = \frac{Q_1}{b}$ $TC(Q_1) = r\frac{Q_1}{b}$; if $MRTS = \frac{W}{r}$ every L*, K* that belongs to the isocost is ok ; *to find TC(Q) repeat the steps ∀Q*	to find the corner $\frac{L}{b} = \frac{K}{a}$ $Q = \frac{L}{b} = \frac{K}{a}$ $L^* = bQ$, $K^* = aQ$ $TC(Q) = wbQ + raQ$

what happens to Q when we increase both L and K?

Constant return to scale: we double the inputs and get double production level ($Q_1 = 2Q$)

Increasing return to scale: doubling the inputs we get more than double the production level ($Q_1 > 2Q$)

Decreasing return to scale: we double the inputs and get less than double the production level ($Q_1 < 2Q$)

total cost (TC) = variable cost (VC) + fixed cost (FC)

variable cost: costs of inputs that vary with the firm's output level

fixed cost: costs of inputs whose use does not vary with the firm's output level

cost curve is equal to the variable cost plus the fixed cost curves

opportunity cost: the cost associated with forgoing the opportunity to employ a resource in its best alternative use

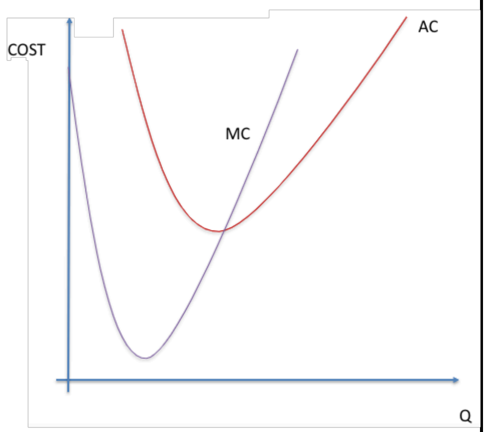
a firm's true economic costs of production consists of both out of pocket expenditures and opportunity costs

average cost: cost per unit of output produced, averaged over all units produced $AC = \frac{C(Q)}{Q}$

marginal cost: extra cost the firm incurs per unit of output added $MC = \frac{\Delta C}{\Delta Q} = \frac{C(Q) - C(Q - \Delta Q)}{\Delta Q} = \frac{\partial C(Q)}{\partial Q}$

AC curve is upward sloping at Q if MC > AC, downward sloping if AC > MC and neither rising nor falling if MC = AC

MC always crosses the AC from below at the efficient scale of production



AC = average variable cost ($AVC = \frac{VC(Q)}{Q}$) + average fixed cost ($AFC = \frac{FC(Q)}{Q}$)

Economies of scale: average cost <u>falls</u> as firm produces more; AC(Q') < AC(Q) for Q' > Q

Diseconomies of scale: average cost <u>rises</u> as firm produces more; AC(Q') > AC(Q) for Q' > Q

A production function can have economies of scale up to a certain output level and then diseconomies of scale.

If **MC < AC**, we are in the range of **economies of scale**.

If **MC > AC** we are in the range of **diseconomies of scale.**

When MC = AC we are at the **most efficient** output level (AC at its lowest).

Microeconomics – first partial

key words

price-taking firm

marginal revenue

supply function of a price-taking firm

law of supply

formulas

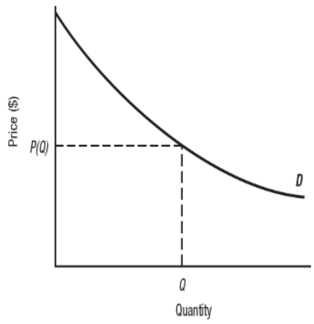
π = TR – TC

max π = PQ – TC(Q)

Profits: firm's revenue – total cost → π = TR – TC

Revenues (TR) = P(Q) · Q

If a firm wants to sell Q units, it needs to set the corresponding price P(Q) from inverse demand function.



In *perfectly competitive markets* price does not depend on single output decisions by a seller: a seller cannot change the market price through his choice of Q.

Price-taking firm: a firm that can sell as much as it wants at P, but nothing at a higher/lower price (perfectly horizontal demand curve).

*How do we find the **profit-maximizing sales quantity** for price-taking firms?*

For a given price P, in perfect competition, to maximize profits a firm chooses a sales output Q to maximize revenues minus costs: max π = PQ – TC(Q)

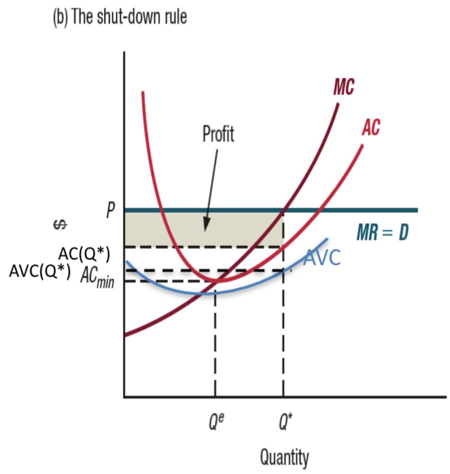
Mathematically: derive function and set it to 0 → [TR (Q) – TC (Q)]' = MR – MC

***Marginal revenue* (MR)**: additional revenue produced by an increase in the output sold.

If MR > MC profits go up if firm sells more.

If MR > MC profits go down if firm sells more.

Optimal choice is a **Q such that MR = MC:**

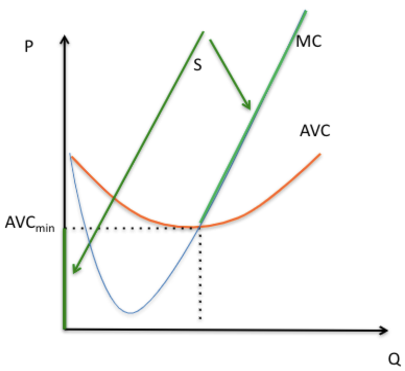


a) QUANTITY RULE: for a price-taking firm, P=MR for all quantities; identify positive sales quantities where **P = MC**. If there is more than one positive sale then determine which produces the highest profit;
b) SHUT-DOWN RULE: check whether the most profitable positive sale quantity results in a greater profit than shutting down.
c) alternative way: check whether revenues are greater than variable costs (PQ* > VC (Q*) or P > AVC (Q*))

Supply function of a price-taking firm: how many units a firm wants to sell at each possible price P: Quantity Supplied = S(P)

From profit maximization, at each price P a firm wants to sell Q such that:

P = MC (Q), and Q = 0 for P < AVC_{min}



green = supply function

If MC of an input increases: AVC and MC curves shift up, supply curve shifts up.

If a fixed cost increases: AVC and MC unaffected, supply curve unaffected.

law of supply: when market price increases the profit-maximizing sale quantity for a price-taking firm never decreases (= supply is always upward sloping)

Microeconomics – first partial

key words

competitive market

market demand

market supply

short-run equilibrium

long-run equilibrium

free entry

aggregate surplus

willingness to pay

avoidable cost of production

formulas

$Q^D_{market} = N \cdot Q_i$

$Q^S_{market} = N \cdot Q_i$

aggregate surplus = tot willingness to pay – tot avoidable cost of production

In a **competitive market** consumers and producers are *price taker*. To analyze the competitive market for a good we need to determine the good's market demand and market supply.

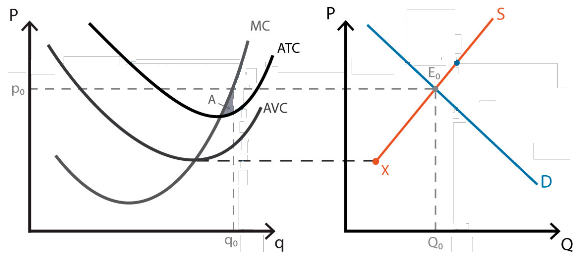
Market demand: horizontal sum of the demands of all individual consumers $Q^D_{market} = N \cdot Q_i$.
If individual demand differs in the short vs long run, market demand will differ as well.

Market supply: horizontal sum of the supply curves of all individual sellers $Q^S_{market} = N \cdot Q_i$.
If individual supply differs in the short vs long run, market supply will differ sa well.

	market supply curve
short run	add up the short-run supply curves of all currently active firms
long run	add up the long-run supply curves of all potential suppliers

short-run equilibrium: $Q^D_{market} = Q^S_{market}$

long-run equilibrium: $Q^D_{market} = Q^S_{market}$



The Q^S_{market} changes in the long run because the MC varies (inputs are all variable → cost changes).

free entry: when entry in the market is unrestricted and technology is freely available to anyone who wishes to start a firm → the **number of potential firms** is **unlimited**.

long-run equilibrium with free entry (very long run): with free entry, the supply curve is horizontal at the level of AVC_{min}. Free entry has **3 implications** for the equilibrium:

1) the equilibrium price equals the minimum average cost: $P^{eq} = AVC_{min}$
2) firms earn zero profit: $\pi_i = 0$
3) each active firm produces at its efficient scale of production MC = AC

$\pi_i = 0$ means that the opportunity costs are being balanced, so there is nothing more profitable the owner can do.

If an economic system works well, it creates *net benefits*: consumers' benefits from the goods they consume exceed the cost of producing them.
To measure this *net benefit* created by the production and consumption of a good we use the ***AGGREGATE SURPLUS*** (total willingness to pay – total avoidable cost of production).

A ***consumer's willingness to pay*** for a particular amount of the good equals the area under the consumer's demand curve, up to that quantity of the good; the total willingness to pay is the sum of all the individuals' willingness to pay.

A ***firm's avoidable cost of production*** equals the area under its supply curve, up to its production level; the total avoidable cost of production is the sum of the firms' avoidable cost of production.